TEMPERATURE FIELD IN A DIELECTRIC MIRROR DURING INTERACTION WITH LASER RADIATION

A. Ya. Balagurov, G. I. Labudin, and V. I. Skobelkin

UDC 536.24

The general problem of the temperature distribution in a flat dielectric mirror during absorption of electromagnetic (EM) waves emitted by a laser in a broad spectrum range is solved.

A flat dielectric mirror (substrate with deposited film) absorbs EM waves, hence the rate of heat absorption $q^+(x, y)$ on the mirror surface is assumed known. According to [1], the mean quantity of heat absorbed per unit time per unit surface of the mirror is determined for plane EM waves by the relationship

$$q^+(\omega, \mathbf{r}) = \frac{\omega}{8\pi}\left[\varepsilon''(\omega)\,\mathbf{EE}^* + \mu''(\omega)\,\mathbf{HH}^*\right].$$

If the radiation spectrum is realized in a broad frequency range (liquid lasers), then the total quantity of heat being absorbed per unit time per unit mirror surface is determined by the formula

$$q^+(\mathbf{r}) = \int_0^\infty f(\omega)\, q^+(\omega, \mathbf{r})\, d\omega,$$

where $f(\omega)$ is the spectral distribution function and $f(\omega)q^+(\omega, \mathbf{r})$ is the spectral density of energy dissipation.

The heat emission of a mirror is determined in a Newton approximation by the equation

$$q^-(\mathbf{r}) = \alpha\,[\overline{T}(\mathbf{r}) - T_0].$$

Under given boundary conditions in the stationary mode of laser operation, the temperature field of the mirror must be determined which satisfies the equation

$$\varkappa \Delta T + q^+ = 0. \tag{1}$$

Integrating (1) along the z coordinate over the mirror thickness, we obtain

$$\varkappa \int_0^{d+h} \Delta T dz + \int_0^{d+h} q^+ dz = \varkappa (d+h)\left(\frac{\partial^2 \overline{T}}{\partial x^2} + \frac{\partial^2 \overline{T}}{\partial y^2}\right) + \varkappa \int_0^{d+h} \frac{\partial^2 T}{\partial z^2} dz + \int_0^{d+h} q^+ dz = 0, \tag{2}$$

where

$$\overline{T}(x, y) = \frac{1}{d+h}\int_0^{d+h} T(\mathbf{r}) dz;\ \varkappa \int_0^{d+h} \frac{\partial^2 T}{\partial z^2} dz = \varkappa \left.\frac{\partial T}{\partial z}\right|_0^{d+h} = -2\alpha(\overline{T} - T_0).$$

Taking into account that $h \ll d$, we obtain from (2)

$$\varkappa \Delta \overline{T}(x, y) - \frac{2\alpha}{d}[\overline{T}(x, y) - T_0] + \overline{q}^+ = 0, \tag{3}$$

where $\overline{q}^*(x, y) = 1/d \int_0^d q^+(\mathbf{r}) dz$.

Let us introduce the dimensionless variables

$$\overline{x} = kx,\ \overline{y} = ky,\ \overline{r} = \sqrt{\overline{x}^2 + \overline{y}^2} = k\sqrt{x^2 + y^2} = kr,$$

where $k^2 = 2\alpha/\varkappa d$. Equation (3) becomes in the variables $\overline{x}$, y

$$\frac{\partial^2 \theta}{\partial \overline{x}^2} + \frac{\partial^2 \theta}{\partial \overline{y}^2} - \theta + q^+(\overline{x}, \overline{y}) = 0, \tag{4}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 4, pp. 724-727, October, 1978. Original article submitted November 3, 1977.

0022-0841/78/3504-1245 $07.50

where

$$\theta(\bar{x}, \bar{y}) = \theta(kx, ky) = \bar{T}(x, y) - T_0 = \tau(x, y);$$

$$q^+(\bar{x}, \bar{y}) = q^+(kx, ky) = \frac{d}{2\alpha}\bar{q}^+(x, y).$$

The general solution of (4) can be written in the form

$$\theta(\bar{x}, \bar{y}) = \iint_{(x', y')} \bar{q}^+(\bar{x}', \bar{y}')\, G(\bar{x}', \bar{y}', \bar{x}, \bar{y})\, d\bar{x}' d\bar{y}',$$

where G is Green's function.

Let the substrate be unbounded. Then Green's function is determined by the equation

$$\ddot{G}_{\xi\xi} + \frac{1}{\xi}\dot{G}_{\xi} - G(\xi) = -\delta(\xi), \tag{5}$$

where $\xi = \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2}$, and by the corresponding boundary conditions.

The general solution of (5) will be

$$G(\xi) = C_1 I_0(\xi) + C_2 K_0(\xi),$$

where I_0 is the cylinder function of zero order and imaginary argument, and K_0 is the zero-order MacDonald function [2]:

$$I_0(\xi) = \sum_{m=0}^{\infty} \frac{\left(\frac{\xi}{2}\right)^{2m}}{(m!)^2}, \tag{6}$$

$$K_0(\xi) = -\left(\ln\frac{\xi}{2}\right) I_0(\xi) + \sum_{m=0}^{\infty} \frac{\xi^{2m}}{2^{2m}(m!)^2}\psi(m+1),$$

$$\psi(m+1) = -C + \sum_{i=1}^{m} \frac{1}{i}, \tag{7}$$

C is the Euler constant equal to $-\psi(1) = 0{,}57721566490\ldots$ Because of the very rapid convergence of the series (6) and (7), a sufficiently good approximation for practice is already obtained for substrate dimensions commensurate with the mirror dimensions. Hence, we limit ourselves to the limit case of an infinite substrate. For $\xi \to \infty$ $I_0(\xi) \to \infty$, hence $C_1 = 0$. Therefore,

$$G(\xi) = C_2 K_0(\xi).$$

We determine the arbitrary constant C_2 from the condition of compliance with the heat balance

$$4\pi\alpha \int_0^{\infty} \xi\tau(\xi)\, d\xi = \left(\frac{4\pi\alpha}{k^2} \int_0^{\infty} \xi K_0(\xi)\, d\xi\right) C_2 = \frac{2\alpha}{k^2},$$

from which

$$C_2 = \left(2\pi \int_0^{\infty} \xi K_0(\xi)\, d\xi\right)^{-1}.$$

Introducing the new constant

$$a = \int_0^{\infty} \xi K_0(\xi)\, d\xi = 1{,}8695\ldots,$$

we obtain Green's function

$$G(\xi) = \left(\frac{1}{2\pi a}\right) K_0(\xi). \tag{8}$$

The general solution of the problem is represented in terms of Green's function as

$$\theta(\bar{x}, \bar{y}) = \left(\frac{1}{2\pi a}\right) \iint\limits_{(x', y')} K_0(\xi)\, q^+(\bar{x}', \bar{y}')\, d\bar{x}' d\bar{y}'.$$

Going over to the x, y coordinates, we obtain

$$\tau(x, y) = \left(\frac{1}{2\pi\varkappa a}\right) \iint\limits_{(x', y')} q^+(x', y')\, K_0(k|r - r'|)\, dx' dy'.$$

The asymptotic behavior of Green's function $K_0(\xi)$ is determined [2] by the formula

$$K_0(k|\mathbf{r} - \mathbf{r}'|) \cong \sqrt{\frac{\pi}{2k|\mathbf{r} - \mathbf{r}'|}} \exp(-k|\mathbf{r} - \mathbf{r}'|). \quad (9)$$

It follows from (9) that the characteristic dimension D outside of which there is practically no temperature field is determined from the inequality

$$D = |\mathbf{r} - \mathbf{r}'| \geqslant \frac{1}{k} = \sqrt{\frac{\varkappa d}{2\alpha}}.$$

The condition upon compliance with which the mirror can be considered as approximately unbounded is expressed by the inequality

$$Q^+ = \iint q^+(x, y)\, dx dy \gg \left| \varkappa d 2\pi r \frac{d\tau}{dr} \right|_{r = D_n},$$

from which we obtain after appropriate calculations and estimates

$$D_n d\varkappa k^3 \sqrt{\frac{\pi}{2kD_n}} \exp(-kD_n) \ll 2\alpha a. \quad (10)$$

For the values D_n, $\varkappa$, α, d existing for mirrors, inequality (10) is satisfied. Therefore, Green's function (8) can be used to compute the temperature field. The results of a computation agree with experiment.

NOTATION

q^+, q^-	are the rates of absorption and heat transmission per unit mirror surface;
x, y	are the coordinates of points of the mirror;
ω	is the spectral frequency of laser radiation;
r, **r'**	are the arbitrary radius-vectors of points of the mirror surface;
ε'', μ''	are the imaginary parts of the complex dielectric and magnetic permittivities;
E, **H**, **E***, **H***	are the vectors of the EM field intensity and their conjugate vectors;
α	is the heat-transfer from the mirror to the external medium referred to unit surface per unit time;
$\bar{T}$	is the average temperature over the mirror thickness;
T_0	is the temperature of the external medium;
$\varkappa$	is the three-dimensional Laplace operator;
d	is the substrate thickness;
C_1 and C_2	are the arbitrary constants;
D_n	is the mirror diameters;
$\delta(\xi)$	is the Dirac function.

LITERATURE CITED

1. L. D. Landau and E. M. Lifshits, Electrodynamics of Continuous Media [in Russian], Gostekhizdat, Moscow (1957).
2. P. M. Morse and H. Feshbach, Methods of Theoretical Physics, McGraw-Hill (1953).